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WORKING PAPER
UCB-ITS-VWP-2014-1

February 2014
On the Impacts of Bus Stops near Signalized Intersections: Models of Car and Bus Delays

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Abstract

Models are formulated to predict the added vehicle and person delays that can occur when a bus stop is located a short distance upstream or downstream of a signalized intersection. Included in the set of models are those that predict the expected delays that cars collectively incur when a bus blocks one of multiple lanes while loading and unloading passengers at the stop. Others in this set predict the expected added delays incurred by the bus due to car queues. Each model is consistent with the kinematic wave theory of highway traffic, as is confirmed through a battery of tests. And each accounts for the randomness in both, bus arrival times at a stop, and the durations that buses dwell there to serve passengers. Though the models are analytical in form, solutions come through iteration. Hence model applications are performed with the aid of a computer.

The applications presented herein show that bus delays can often be shortened by placing the bus stop downstream of its neighboring signalized intersection, rather than upstream of it. In contrast, car delays are often shortened by placing the stop some distance upstream of the intersection, rather than downstream. We further show how exerting a measure of control on bus arrivals can further enhance these benefits to cars without further delaying the buses. The models are also used to assess the net person delays collectively incurred by car- and bus-travelers.

Keywords: near-side and far-side bus stops; kinematic wave theory; car delays; bus delays; bus holding

1. Introduction

Curbside bus stops are commonly located short distances from signalized intersections, in part to facilitate passenger transfers between perpendicular bus lines (Fitzpatrick et al, 1996). The bus stop may reside upstream of its neighboring intersection (a so-called near-side stop) or downstream of it (a far-side stop); see Fig. 1. In either case, buses may occupy a travel lane while dwelling at the stop to load and unload passengers. A dwelling bus can therefore become a bottleneck that constrains car flows near the intersection. This can cause car queues to expand, which can further delay the buses as well as the cars.

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The above concerns have long engendered debates on where best to locate a bus stop relative to its nearby intersection. Some studies have found in favor of far-side stops (e.g. Terry and Thomas, 1971). Others have reached the opposite conclusion (e.g. Fitzpatrick et al, 1997). The matter has yet to be settled, in part because observational studies, like those cited above, and studies based on simulation or meta-heuristic search algorithms (Gibson, 1996; Joyce and Yagar, 1990; Wong et al, 1998; Zhao et al, 2007; 2008; Moura et al, 2011), have by necessity focused on relatively small numbers of select cases.

The literature also includes a few bus-stop models that are analytical in form. These would presumably allow for evaluation over a broader range of cases, except that these models have limitations. For example, some of the best known of these analytical models are in the *Highway Capacity Manual* (TRB, 2000),¹ and these have been found to be deficient in various ways (Gibson, 1996; Holt, 2004; Gu et al, 2011). Yet another of the existing analytical models is limited in that it treats bus stops as if they were isolated from car traffic (Ghoneim and Wirasinghe, 1980).

The literature also includes a more realistic analytical model that predicts the delays that cars can impart to buses (Furth and SanClemente, 2006). However, that model is formulated for cases in which stops are placed in so-called bus bays, so that dwelling buses are removed from travel lanes and do not impede cars. Hence that model says nothing about the car delays that can occur in the absence of bus bays.

In response to these limitations, some of the present authors have more recently explored one particular impact that buses can have on cars, that being: the residual queues of cars that can form at an intersection approach when its near-side stop is occupied by a bus (Gu et al, 2013). That work assumed that the approach has a fixed car demand that is lower than the capacity of the restriction created by the dwelling bus. Still, the discharge flow of queued cars may be constrained by that bus when the traffic signal turns green. The analytical models formulated for those circumstances can be used to determine where to place a near-side stop, either to limit the number of signal cycles that a residual car queue will persist, or to prevent the formation of these queues entirely.

In that same earlier work, the authors also developed and theoretically tested a strategy that postpones the arrivals of some buses at a near-side stop. This “bus holding” was done in such ways as to reduce further the occurrence of residual car queues, without delaying buses over the longer run.

¹ Some of these same models are now furnished in the *Transit Capacity and Quality of Service Manual* (Kittleson & Associates et al, 2013), which supplants discussion of transit systems in later editions of the *Highway Capacity Manual*. 
The above-cited models of Gu et al (2013) were derived as per the logic of the simplified theory of kinematic waves (Newell, 1993), and are therefore consistent with the physics of real highway traffic. However, that earlier work made no attempt to predict how the selection of a stop’s location might affect the delays on the approach, whether incurred by the cars or by the buses, or by the occupants of either vehicle class. Moreover, that work said nothing about far-side stops.

The present paper fills those holes. Rather than predicting the incidence of residual car queues, our new models predict the expected additional delays incurred: collectively by car traffic as a result of each dwelling bus; and by each bus, as a result of the car queues. Our focus on delay makes sense since it is a common metric for assessing quality of service (e.g. Kittleson & Associates et al, 2013; Gu et al, 2014). Moreover, we now furnish models for far-side stops as well as for near-side ones. A model of expected car delay is also developed for cases in which bus holding is deployed at near-side stops. We shall use these models to unveil circumstances under which near-side stops are preferable to far-side ones, and vice-versa.

Like those of Gu et al (2013), the present models are formulated in accord with the simplified kinematic wave theory of Newell (1993). This makes sense given that we will compare stop types by assuming a steady-state demand for cars, as is commonly done in transport-planning applications. And like those of Gu et al, the present models are formulated for under-saturated intersection approaches, where a dwelling bus can constrain cars only as they discharge from queues during green times.

Though the present models are analytical in form, iterative solutions are required. Moreover, the models compute expectations that account for the randomness in both: the bus arrival times at a stop; and the durations that buses dwell there to serve passengers. Solutions therefore require the aid of a computer, and more will be said on this matter in due course.

Before proceeding further, we define our notation and assumptions. Some of these are borrowed from Gu et al (2013).

1.1 Notation and assumptions

We denote: $d$ as the distance between a stop and its neighboring signalized intersection; $L_C$ as the fixed length of the signal cycle; $G$ as the fixed green time each cycle; and $g = G/L_C$ as the green ratio. As per Newell (1993): we assume that a triangular-shaped fundamental diagram describes the states of car traffic on the approach; and we perform analyses using a moving coordinate system for time.

An example fundamental diagram and set of car states are shown in Fig. 2. The point labeled $C$ on the diagram is the state when cars discharge at the capacity of the road approach, $Q$. Point $J$ is the jam state with a car density of $Q/w$, where $w$ is the backward wave speed in a car queue. Point $A$ is the state of freely flowing cars with a stationary inflow, $q$. Points $B$ and $D$ are the car states downstream and upstream of the bottleneck created by a bus dwelling in a travel lane, whether at a near- or far-side stop. The capacity of that bottleneck is denoted $Q_B$. The speeds of the waves that separate the distinct car states, denoted $w_{AJ}$, $w_{AD}$, and $w_{BJ}$ in Fig. 2, can be expressed as functions of the known inputs $Q$, $q$, $Q_B$ and $w$:

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2 Notwithstanding the word “simplified,” the theory has been shown to faithfully replicate highway traffic in real settings (Cassidy and Mauch, 2001; Cassidy, 1998).

3 For simplicity, turning traffic on the approach is ignored; i.e., $q$ is the through-moving car inflow (only).
\[
\begin{align*}
\omega_{AD} &= \frac{w(Q_B-q)}{Q-Q_B}; \\
\omega_{AJ} &= \frac{Wq}{Q}; \\
\omega_{BJ} &= \frac{WQ_B}{Q}.
\end{align*}
\]

As previously noted, car demand on the intersection approach is assumed to be low enough to promote under-saturated conditions, i.e., \(q \leq gQ\). As a result, a bus dwelling at a curbside stop will not restrict the un-queued inflow of cars, i.e., \(q \leq Q_B\).\(^4\) It is further assumed that a bus’ dwell time, \(S\), is a random variable bounded from above by \(L_C\); and that bus arrivals are random and can be treated independently, due to varied but sufficiently large headways (i.e. inter-arrival times) between them.

![Fig. 2 Fundamental diagram (in moving-time coordinates) for the street upstream or downstream of the intersection](image)

**2. Delay models for near-side stops**

To begin, consider the time-space diagram in Fig. 3. It displays a periodic queuing pattern that cars might exhibit on an intersection approach in the absence of a dwelling bus. The circled letters \(J\), \(C\), and \(A\) denote the car states previously shown in Fig. 2. The slopes of the interfaces between any two neighboring states

\(^4\) This assumption simplifies our analysis, and is valid in most real cases. To see why, we note that in many instances \(Q_B \approx \frac{n-1}{n} Q\), where \(n\) is the number of lanes on the subject approach. Thus we have \(Q_B \geq \frac{Q}{2}\). (In real settings, buses often pull aside the street to load/unload passengers so that cars have more road space when they pass the dwelling buses. This will further increase \(Q_B\); see Tyler et al, 2002). At the other extreme, \(q\) tends to be bounded by \(gQ\) to avoid over-saturation, where the green ratio, \(g\), is often less than \(1/2\). Hence we commonly find that in real settings, \(q \leq \frac{Q}{2} \leq Q_B\).
are the wave speeds $w_A$ and $w$.\footnote{Since a moving-time coordinate system is used, interfaces that travel at the free-flow vehicle speed appear in Fig. 3 to travel at infinite speed, as described in Newell (1993).} We specify time 0 in Fig. 3 to be the end of a red phase, and $t_e = \frac{(1-g)L_C}{q-g}$ to be the time when a car queue would fully dissipate when not impeded by a dwelling bus.

The lightly-drawn, horizontal dashed line in Fig. 3 marks the location of a near-side bus stop that resides distance $d$ from the intersection. Were a bus to have entered the scene, its arrival time at the stop, $t_a$, could have only taken values between $\frac{d}{w}$ and $gL_C + \frac{d}{w_A}$, the interval circled by a dotted ellipse in the figure. This is because a bus cannot arrive at the stop while it is engulfed by a queue of jammed cars.

Having dispensed with the above preliminaries, a model will next be derived for the added delay collectively imposed on cars by a near-side dwelling bus. For clarity of exposition, a case-specific model is first derived for select values of $t_a$ and bus dwell time, $S$. A generalized car-delay model is thereafter developed by taking expectations over all feasible $t_a$ and $S$. The model’s consistency with kinematic wave theory (Lighthill and Whitham, 1955; Richards, 1956) is then tested against simulation.

A model of added bus delay due to the presence of car queues is similarly derived in Section 2.2. A case-specific model for select $t_a$ and $S$ is formulated first. A generalized bus-delay model follows from that.

![Fig. 3 Time-space diagram for a near-side stop absent the influence of a dwelling bus](image)

2.1 Car delays imposed by a dwelling bus

Consider now the time-space diagram in Fig. 4a, where a bus’ trajectory is plotted as a piecewise-linear, boldly-drawn arrow. Note how in this case the bus arrives at the near-side stop between times $\frac{d}{w}$ and $t_e$. Further note how the bus continues to dwell there even after the cars queued downstream of the stop have discharged, i.e., $t_a + S \geq L_C + \frac{d}{w}$.

The spatiotemporal car states that result from the dwelling bus are also shown in Fig. 4a. Note that after arriving at the stop at time $t_a$, the dwelling bus constrains the discharging cars that pass it.
Recall that this constrained queue discharge flow is \( Q_B \). The constrained rate persists for a duration \( T_1 = gL_C - t_a \). At the start of the next green phase, cars queued downstream of the occupied bus stop discharge at capacity \( Q \) (the flow corresponding to state C in Fig. 2). This capacity flow persists only for a duration \( \frac{d}{w} \), after which time the cars that were queued upstream of the dwelling bus begin discharging into the intersection at the constrained rate \( Q_B \). This, in turn, persists for duration \( T_2 = t_a + S - L_C - \frac{d}{w} \), after which time the bus departs the stop and car discharge flow recovers to \( Q \).

Fig. 4 A typical dwelling bus at a near-side stop: (a) time-space diagram; (b) diagram of cumulative car counts
Now envision Fig. 4a with a third axis for vehicle count, as described in Makigami et al (1971). We thusly obtain the cross-sectional view of car states at the entrance to the intersection; see the cumulative count curve of entering cars, labeled $D(t)$ in Fig. 4b. The slopes of that curve are the car discharge flows. A second cumulative count curve, labeled $D'(t)$ in Fig. 4b, is obtained for the case in which car flows are not affected by a dwelling bus. Thus, the added delay imposed across all cars by the bus, $W_C$, is given by the shaded area between $D(t)$ and $D'(t)$ in the figure:

$$W_C = \int_0^\infty (D'(t) - D(t)) dt.$$  \hspace{1cm} (2)

The $W_C$ can thus be computed by geometry given $\dot{D}(t)$ and $\dot{D}'(t)$, the slopes of the piecewise linear curves in Fig. 4b. These derivatives can be obtained directly from that figure as follows:

$$\dot{D}'(t) = \begin{cases} Q, & \text{if } nL_C \leq t < nL_C + t_e \\ q, & \text{if } nL_C + t_e \leq t < (n + g)L_C \quad (\text{for } n = 0,1,2, \ldots), \\ 0, & \text{if } (n + g)L_C \leq t < (n + 1)L_C \end{cases}$$  \hspace{1cm} (3)

$$\dot{D}(t) = \begin{cases} Q, & \text{if } 0 \leq t < t_a \\ Q_B, & \text{if } t_a \leq t < gL_C \\ 0, & \text{if } gL_C \leq t < L_C \\ Q, & \text{if } L_C \leq t < L_C + \frac{d}{w} \\ Q_B, & \text{if } L_C + \frac{d}{w} \leq t < t_a + S \\ Q, & \text{if } t_a + S \leq t < (1 + g)L_C \quad \ldots \end{cases}$$  \hspace{1cm} (4)

The omitted portions of (4) represented by the ellipsis can be constructed iteratively, cycle by cycle. In each cycle, the following rule applies: if $\dot{D}'(t) - D(t) > 0$ and $t$ falls in a green phase, $\dot{D}(t) = Q$; otherwise, $\dot{D}(t) = \dot{D}'(t)$. The iteration ends when $D(t)$ and $D'(t)$ re-converge.

**General solution**

We take the expected value of $W_C$, denoted as $E(W_C)$, for all feasible $t_a$ and $S$. For an arbitrary pair of $t_a$ and $S$, the $W_C$ is still given by (2) and $\dot{D}'(t)$ is still obtained as in (3). However, the $\dot{D}(t)$ is now as follows:

$$\dot{D}(t) = \begin{cases} Q, & \text{if } 0 \leq t < t_a \\ Q_B, & \text{if } t_a \leq t < t_a + T_{B1} \\ 0, & \text{if } gL_C \leq t < L_C \\ Q, & \text{if } L_C \leq t < L_C + \frac{d}{w} \\ Q_B, & \text{if } L_C + \frac{d}{w} \leq t < L_C + \frac{d}{w} + T_{B2} \quad \ldots \end{cases}$$  \hspace{1cm} (5)

---

*The $D'(t)$ curve was constructed with the aid of the virtual cumulative count curve, where the latter is the dashed line labeled $V(t)$ in Fig. 4b; e.g. see Daganzo (1997). The $V(t)$-curve describes how cars would have entered the intersection in the absence of any delays, whether imposed by a dwelling bus, the traffic signal or something else.*
where $T_{B1}$ and $T_{B2}$ denote the green durations when car flows are constrained by a dwelling bus: $T_{B1}$ occurs in the cycle that coincides with the bus’ arrival, and $T_{B2}$ in the cycle that follows immediately thereafter.

The $T_{B1}$ can take the minimum of: (i) the bus dwell time, $S$; (ii) the time interval from $t_a$ to the end of the green phase in that same cycle, $gL_C - t_a$; and (iii) the time needed to fully discharge the cars queued upstream of the stop at the constrained flow $Q_B$, i.e., $(t_e - t_a) \frac{Q-q}{Q_B-q}$. However, if one of the above three is negative, then $T_{B1} = 0$. Thus,

$$T_{B1} = \max \left\{ 0, \min \left\{ S, gL_C - t_a, (t_e - t_a) \frac{Q-q}{Q_B-q} \right\} \right\}. \quad (6a)$$

In the very next cycle, $T_{B2} > 0$ only if $t_a + S - L_C - \frac{d}{w} > 0$, i.e., if the dwelling bus continues to impede discharging cars in that next cycle. In that case, one may imagine the final portion of $S$ that extends beyond $L_C + \frac{d}{w}$ as belonging to a new bus that arrives at time $t_a' = \frac{d}{w}$, and that dwells for duration $S' = t_a + S - L_C - \frac{d}{w}$. Thus by replacing $t_a$ and $S$ in $(6a)$ with $t_a'$ and $S'$, we have:

$$T_{B2} = \max \left\{ 0, \min \left\{ t_a + S - L_C - \frac{d}{w}, gL_C - \frac{d}{w}, (t_e - \frac{d}{w}) \frac{Q-q}{Q_B-q} \right\} \right\}. \quad (6b)$$

The portions of $\dot{D}(t)$ omitted in $(5)$ can again be constructed iteratively following the same rule described immediately below $(4)$.

**Tests**

Its iterative form notwithstanding, our general-solution method faithfully replicates the kinematic wave theory of highway traffic (Lighthill and Whitham, 1955; Richards, 1956). We illustrate this by comparing for numerous scenarios the $E(W_C)$ from our formulas against the outcomes from simulation. In every scenario: $Q = 1$ vehicle/s, $Q_B = 0.5$ vehicle/s, $L_C = 90$ s, $g = 0.5$, $w = 7$ m/s and the $S$ was always uniformly distributed over the range $35 s \leq S \leq 55$ s.

We assume that a bus arrival time as measured upstream of a car queue is uniformly distributed over a signal cycle. However, a portion of those buses will be obstructed by car queues before reaching the stop; e.g. see the bus trajectory in Fig. 4a. Thus, $t_a$ will follow a distorted, uniform-like distribution with a probability density function given by:

$$f_{t_a}(t) = \begin{cases} 0, & \text{if } t \in \left[\frac{dq}{lwq} - (1 - g)L_C, \frac{d}{w}\right] \\ \frac{1}{L_C} \cdot \frac{Q}{q}, & \text{if } t \in \left[\frac{d}{w}, t_e\right] \\ \frac{1}{L_C}, & \text{if } t \in \left[t_e, gL_C + \frac{dq}{lwq}\right]. \end{cases} \quad (7)$$

Our formulas were coded in a MATLAB program. For each scenario, the code estimated the $E(W_C)$ by randomly generating 10,000 sets of $t_a$ and $S$ from their specified distributions, and computing the average of the 10,000 resulting $W_C$. 

8
The simulations were performed using the Cell Transmission Model (CTM), since it is known to approximate kinematic wave theory; see Daganzo (1994). For parity, the outcome for each scenario was taken to be the average of 10,000 simulated instances, where the \( t_a \) and \( S \) were again randomly generated for each instance.

Comparisons for an under-saturated case \( \left( \frac{q}{Q} = 0.4 \right) \) and for a nearly-saturated one \( \left( \frac{q}{Q} = 0.47 \right) \) are presented in Fig. 5. Note the near-perfect agreements for smaller values of \( d \). The small differences that do arise for \( d < 100 \text{m} \) can be attributed to the CTM’s approximate nature: its bus arrival times and the completions of bus dwell times occur within the resolution of a time step, which is 1s in the present case.

![Diagram](image)

Fig. 5 Comparison of stop location versus expected additional car delay at near-side stops for analytical model and simulation.

Note too how differences between analytical and simulated outcomes grow with increasing values of \( d \). These growing differences can also be attributed to the discretization of time and space in the CTM. This discretization leads to backward-moving shocks that soften as they travel upstream, which creates small discrepancies between the CTM and kinematic wave theory. As an example, consider the colored shadings in Fig. 6a, which is the output of a CTM simulation; in this figure, lighter shadings represent larger cell densities. The solid lines overlaid onto this CTM output represent the theoretically predicted shockwaves from kinematic wave theory for the same scenario. Notice that the transition between states J and C are not as sharply defined in the CTM output as the theoretical predictions—these shocks become softer (i.e., become spread out over a longer period of time) as they move further upstream of the intersection. The presence of these softer shocks dilutes the impact of buses that are dwelling near the

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Note that this figure is plotted using a traditional coordinate system (i.e., regular time units) as the CTM cannot directly replicate the moving-time coordinate system.
time that the shock reaches the stop location. For an illustration, consider the analytically derived time-space diagrams (in moving-time coordinates) shown in Fig. 6b and Fig. 6c, which present a bus that is already dwelling at the stop when the theoretically predicted backward shockwave between the J and C states arrives at the stop. When the shock between these two states is sharp (Fig. 6b), vehicles will discharge through the intersection at the maximum rate $Q$ until some time $t_c$, after which discharge rate will be reduced to $Q_B$. However, this period of maximum discharge rate increases by a small amount when the shock is softened (represented by the shaded region in Fig. 6c). This additional period of maximum discharge increases with the temporal width of the softened shock (i.e., the distance of the shock from the intersection), and results in less vehicular delay caused by the bus. For these reasons, the CTM simulations under-predict delays at near-side stops, and the amount of the under-prediction increases with the distance of the bus stop from the intersection. Tests have confirmed that the difference created by this effect disappears as the clock tick approaches zero, but such small clock ticks increase the simulation runtime exponentially. Still, even the largest discrepancies evident in Fig. 5 are within 40 vehicle-s for $\frac{q}{Q} = 0.4$, and 120 vehicle-s for $\frac{q}{Q} = 0.47$. The figure thus illustrates our method’s consistency with kinematic wave theory.\(^8\)

\(^8\) Moreover, the additional car delays are remarkably insensitive to moderate levels of stochastic variation in vehicle arrivals at the intersection. Simulation tests were performed in Gu et al (2013) in which the car arrival rates within 0.5-s clock ticks were drawn from a uniform distribution between 75% and 125% of its mean value, $q$. The resulting delays showed very little deviation from the deterministic CTM results displayed in Fig. 7 of that reference.
2.2 Bus delays imposed by car queues

A bus will incur added delay at a near-side stop if (i) a car queue impedes the bus from reaching the stop, as in Fig. 4a; and if after loading and unloading its passengers at the stop (ii) a car queue temporarily blocks the bus from passing through the intersection downstream. We define these impacts to be the bus’ pre- and post-loading delays, $W_{B, pre}$ and $W_{B, pos}$, respectively; and the bus delay to be $W_B = W_{B, pre} + W_{B, pos}$.

Consider the time-space diagram in Fig. 7. From its geometry we obtain:

\[ W_{B, pre} = (1 - g)L_C - t_a \frac{Q-q}{q}, \]  
\[ (8a) \]

\[ W_{B, pos} = \left(1 - \frac{gq}{Q}\right)L_C - \left(t_a + S\right) \frac{Q-q}{q}. \]  
\[ (8b) \]
General solution

The pre-loading delay for the general case is simply:

\[ W_{B,\text{pre}} = \max \left\{ 0, (1 - g)L_c - t_a \frac{Q - q}{q} \right\}. \]  

(9)

The post-loading delay can be calculated as follows:

\[
W_{B,\text{pos}} = \begin{cases} 
0, & \text{if } t_a + S \leq gL_c \text{ or } L_c + \frac{d}{w} \leq t_a + S \leq (1 + g)L_c \\
\max \left\{ 0, \min \left\{ L_c \left( 1 - \frac{gq}{Q} \right) - (t_a + S) \frac{Q - q}{Q}, L_c + \frac{d}{w} - (t_a + S) \right\} \right\}, & \text{if } gL_c \leq t_a + S \leq L_c + \frac{d}{w}, \ t_a \leq t_e \text{ and } (q - gQ_B)L_c \leq t_a(Q - Q_B) \\
\max \left\{ 0, \min \left\{ \frac{-t_a(2Q - Q_B - q) - S(q - q) + L_c(Q + (1 - g)q - gQ_B)}{Q}, L_c + \frac{d}{w} - (t_a + S), \right\} \right\}, & \text{if } gL_c \leq t_a + S \leq L_c + \frac{d}{w}, \ t_a \leq t_e \text{ and } (q - gQ_B)L_c > t_a(Q - Q_B) \\
\min \left\{ L_c \left( 1 - \frac{gq}{Q} \right) - (t_a + S) \frac{Q - q}{Q}, L_c + \frac{d}{w} - (t_a + S) \right\}, & \text{if } t_a > t_e \text{ and } gL_c \leq t_a + S \leq L_c + \frac{d}{w} \\
L_c \left( 2 - \frac{(1 + g)q}{Q} \right) - (t_a + S) \frac{Q - q}{Q}, & \text{if } t_a > t_e, t_a + S > (1 + g)L_c, \text{ and } (q - gQ_B)L_c \leq \frac{d}{w}(Q - Q_B) \\
\min \left\{ \frac{-\frac{d}{w}(Q - Q_B) - (t_a + S)(q - q) + L_c(2Q - gQ_B)}{Q}, 2L_c + \frac{d}{w} - (t_a + S), \right\}, & \text{if } t_a > t_e, t_a + S > (1 + g)L_c, \text{ and } (q - gQ_B)L_c > \frac{d}{w}(Q - Q_B). \end{cases} \]

(10a) \quad (10b) \quad (10c) \quad (10d) \quad (10e) \quad (10f)
Equation (10a) corresponds to the case in which a bus’ dwell time ends within a green period and the bus discharges through the intersection without delay. For (10b-d) the bus’ dwell time does not extend into the second green period. In (10b) the dwelling bus creates a car queue in the first signal cycle, but the queue does not carry-over to the second cycle. In (10c) the dwelling bus creates a car queue in the first cycle that carries-over to the second. In (10d) the dwelling bus does not create any additional car queue at all, but the bus might still be obstructed by the car queue created by a red signal. In (10e) and (10f) the bus’ dwell time extends into the second green period, and the portion of the dwell time beyond \( L_C + \frac{d}{w} \) can be imagined to be the trajectory of a new bus where (10b) and (10c) are employed after replacing \( t_a \) with \( t_a' = \frac{d}{w} \) and \( S \) with \( S' = t_a + S - L_C - \frac{d}{w} \).

3. Far-Side Stops

The models for far-side stops follow the logic used in Section 2. The discussion can therefore be shortened.

3.1 Car Delays

The \( E(W_C) \) is given by (2) just as before, where \( D'(t) \) is again determined by integrating (3), and \( D(t) \) by integrating \( \dot{D}(t) \). This time the latter is given by:

\[
\dot{D}(t) = \begin{cases} 
Q, & \text{if } 0 \leq t < t_a \\
Q_B, & \text{if } t_a \leq t < t_a + T_{C1} \\
Q, & \text{if } t_a + T_{C1} \leq t < t_a + T_{C1} + T_{Q1} \\
\dot{D}'(t), & \text{if } t_a + T_{C1} + T_{Q1} \leq t < L_C \\
Q_B, & \text{if } L_C \leq t < L_C + T_{C2} \\
\end{cases}
\]

where \( T_{C1} \) and \( T_{C2} \) are defined identically to \( T_{B1} \) and \( T_{B2} \), respectively (see again Section 2.1), though the computations are distinct. The first of these is given by

\[
T_{C1} = \max \left\{ 0, \min \left\{ S, (t_e - t_a) \right\} \right\}; \quad (12a)
\]

where: \( (t_e - t_a) \) is the time needed if the car queue that extends from the occupied bus stop can be fully discharged at rate \( Q_B \) before the first green phase ends; \( \frac{L_C q - t_a Q}{Q_B} \) is the time needed to discharge that car queue if it cannot be fully served in the first green phase; and \( gL_C + \frac{d(q-Q_B)}{w q_B} - t_a \) is the time needed to discharge only the cars queued between the occupied bus stop and the upstream intersection.

The interval \( T_{C1} \) is followed by a period in which cars can discharge at capacity \( Q \), denoted as \( T_{Q1} \); see the third equation in (11). Its duration is obtained by knowing both, the signal phase displayed when interval \( T_{C1} \) comes to an end, and the number of queued cars that persist downstream of the intersection at that precise time. Hence,
\[ T_{Q1} = \begin{cases} 
\max \left\{ 0, t_e - t_a - S \frac{Q_B - q}{Q - q} \right\}, \\
\max \left\{ 0, \frac{L_C q - S Q_B}{Q} - t_a \right\}, \\
\max \left\{ 0, g L_C - t_a - S \left( g L_C + \frac{d(Q - q)}{w Q_B} - t_a - S \right) \frac{Q_B}{Q} \right\}, \\
\max \left\{ 0, \frac{L_C q - S Q_B}{Q} - t_a \right\}, \\
\end{cases} \] (12b)

The \( T_{C2} \) is obtained by imagining a bus that arrives at \( t'_a = L_C \), and that dwells for a duration \( S' = t_a + S - L_C \). From our assumption that \( S \leq L_C \), we have \( S' \leq g L_C \), meaning that the dwell time of the imaginary bus will never extend beyond the green period. Thus,

\[ T_{C2} = \max \left\{ 0, \min \left\{ t_a + S - L_C, t_e \frac{Q - q}{Q_B - q} \right\} \right\}. \] (12c)

The segments of \( \dot{D}(t) \) that are omitted in (11) are again constructed in iterative fashion knowing that: (i) \( \dot{D}(t) = Q \), if \( D'(t) - D(t) > 0 \) and \( t \) falls within a green phase; and (ii) \( \dot{D}(t) = \dot{D}'(t) \) otherwise.

**Tests**

Comparisons between our analytical predictions and those from the CTM simulation model are made in Fig. 8. Note the near-perfect agreements for the entire range of \( d \). These agreements occur because forward-moving shocks from the signal to the far-side bus stop are not smoothed-out in the CTM, unlike the backward-moving waves that would arrive at near-side stops. Thus, the impacts of dwelling buses at far-side stops are captured more exactly in the CTM than the impacts of dwelling buses at near-side stops.
3.2 Bus delay

Since the car queues that form behind an occupied far-side stop will not affect the dwelling bus, the only source of added bus delay is the car queue that can form at the signalized intersection upstream of the stop. Thus if bus arrivals are uniformly distributed over the signal cycle, the expected added bus delay becomes:

\[ E[W_B] = \frac{(1-g)^2 Q L c}{2(Q-q)}. \]  

(13)

4. Numerical Analysis

The analytical models are now used to estimate delays imposed by near- and far-side stops, and to draw comparisons across these two stop types under various scenarios. Car delays are compared in Section 4.1; bus delays in 4.2; and traveler (i.e. person) delays in 4.3.

4.1 Added Car Delays

Consider first the effects of \( d \). Fig. 9a reveals that when the approach is under-saturated, a near-side stop is less damaging to car traffic than is a far-side one, whatever the \( d \). Note how the near-side stop’s negative impacts steadily diminish as \( d \) increases, and disappear for \( d > 200\text{m} \). No surprise here: when a stop is placed further upstream of the signalized intersection, more of the green time can be used to discharge the car queue at maximum rate \( Q \). The figure also reveals that the damage done by far-side stops also diminishes with larger \( d \), though not as dramatically. This is also consistent with intuition: all
else equal, a larger $d$ means that more cars will be queued downstream of the intersection, and thus more cars can discharge past the dwelling bus during the signal’s red time.

Things change a bit when the approach is nearly saturated, as revealed in Fig. 9b. Cars still benefit from larger $d$, but now we see that for moderate distances ($50m < d < 200m$) cars benefit more from a far-side stop than from a near-side one. This too reflects the benefits of storing long car queues downstream of the intersection.

Details of bus dwell time also exert an influence. To illustrate, consider a $S$ that is uniformly distributed with $\Delta S = S_{MAX} - S_{MIN} = 20s$. The models show that $E(W_C)$ increases with increasing $E(S)$, whether the stop is a near- or far-side one; see Fig. 10a and b.

More interestingly, Fig. 10c furnishes comparisons across stop types for ranges of $E(S)$ and $d$. The contour lines in that figure show the differences (in vehicle-s) in the $E(W_C)$ across the two stop types. Positive values denote cases in which far-side stops produce lower added delays than do near-side ones.

We note that Fig. 10c was constructed for a near-saturated approach ($\frac{q}{Q} = 0.47$), the condition under which far-side stops are more favorable to cars. Still, we see that cars are made better off by a far-side stop only when $E(S)$ is large and $d$ is of moderate size.

![Fig. 9](image-url) Comparisons of expected additional car delays at near-side and far-side stops where $Q = 1, Q_B = 0.5, S_{MIN} = 50, S_{MAX} = 70$: (a) $\frac{q}{Q} = 0.4$; (b) $\frac{q}{Q} = 0.47$
Fig. 10 Car delay contours ($q = 0.47, \Delta S = 20$ sec) of: a) near-side stops without holding; b) far-side stops; c) difference between near-side without holding and far-side stops
4.2 Bus Delays

Recall from (13) that the car queues that form at the signalized intersection are the only sources of \( E(W_B) \) when a far-side stop is used. In that case, \( \frac{q}{Q} \) will influence added bus delay, but \( d \) and \( S \) will not.

Fig. 11 was constructed with the above in mind. It presents the differences between the \( E(W_B) \) at near- and far-side stops, such that positive values denote cases in which the latter stop type is less disruptive to buses. Note that both under- and near-saturated cases are considered, though the figure reveals that these distinctions are of little consequence. The figure further reveals that for either \( \frac{q}{Q} \), the differences in bus delays are greatest when \( d \) is small. For \( d < 50 \text{m} \), a bus approaching a near-side stop runs greater risk of being blocked by car queues, and of being further delayed by these queues once bus passengers have been loaded and unloaded.

To illustrate the influence of bus dwell time, the contours in Fig. 12 display the differences between near- and far-side \( E(W_B) \) for select ranges of \( E(S) \) and \( d \). All contours in the figure display positive values, meaning that near-side stops impart greater delays to buses. The only exception occurs when \( E(S) \) or \( d \) is sufficiently large, in which cases the car queues that form during red phases are the only sources of added bus delays, and no distinctions between near-and far-side stops exist in this regard.

The figure further reveals that for a \( d < 200 \text{m} \), the delay differences initially increase, and then ultimately decrease, as \( E(S) \) increases. A dashed vertical line is added to Fig. 12 to highlight this pattern. We also see that for a given \( d < 200 \text{m} \) within that range, near-side bus delays reach maximums at intermediate values of \( E(S) \). The figure’s solid arrow maps this latter pattern. It is due to the physics that occur at near-side stops. To explain, we note that at near-side stops: (i) a bus with a short dwell time can often discharge through the intersection during the same green phase encountered upon arriving at the stop; and (ii) a bus with a long dwell time can serve boarding and alighting passengers during a red phase, when discharging into the intersection would have been impossible anyway.

![Average delay difference per bus (s)](image)

**Fig. 11** Bus delay difference (near-side minus far-side) for \( \Delta S = 20s \)
4.3 Traveler Delays

Having found that near-side stops are often favorable to cars (see again Section 4.1) and that far-side stops are nearly always favorable to buses (Section 4.2), we now explore the added delays collectively imparted to the people who travel aboard these distinct vehicle classes. The task requires only that we weigh the added delays for each vehicle class by their number of onboard occupants.

![Graph showing bus delay difference versus $E[S] \times d$ ($\frac{Q}{Q_i} = 0.4, \Delta S = 20s$)](image)

For illustration, Fig. 13 furnishes expected traveler delays as functions of $d$ for the case in which: $\frac{Q}{Q_i} = 0.4$; bus onboard occupancy, $O_B = 40$; and car occupancy, $O_C = 1.5$. Note that when $d$ is small, far-side stops trump near-side ones. This is due to the substantially smaller delays that buses enjoy at far-side stops with small $d$, as previously revealed in Fig. 11. (Recall that expected car delays for both stop types are comparable when $d$ is small; see again Figs. 9a and b.) Thus if a stop is to be placed very close to an intersection, a far-side one may be the best way to go.

However, a near-side stop can sometimes be the better option for travelers if one is willing to place the stop sufficiently far from the intersection. Recall that when $d$ is large, a near-side stop can be much more favorable to cars (see again Fig. 9a), while the impacts of both stop types on buses are similar (see again Fig. 11). We see from Fig. 13 that this can translate to less collective delays to travelers, despite the small $O_C$.

Of course, the net impact of stop placement will depend upon the relative occupancy numbers that are onboard the two vehicle classes. To illustrate this point, the curve in Fig. 14 bounds the combinations of $d$ and occupancy ratio, $O_B/O_C$, for which one stop type is preferable to the other in terms of the expected added traveler delay. Notice the expanded range of $O_B/O_C$ for which near-side stops can be the better option when $d$ can be made suitably large.

The benefits of a near-side stop can be made even greater by exerting a measure of control on select buses as they approach the stop. This matter is explored in the following section.
5. Bus Holding

Consider now a strategy in which drivers of select buses are instructed (e.g. via onboard communication devices) to slow in advance of a proximate near-side stop. The only buses to be held in this fashion are those projected to otherwise (i) impede discharging car flows during green times; and (ii) become themselves delayed by car queues when departing from the stop. Gu et al (2013) shows how by judiciously selecting candidate buses, their arrivals at near-side stops can be deferred in time so as to reduce the occurrence of residual car queues at the intersection, without delaying those buses over the longer run. We now explore how this bus-holding can reduce the added delays to cars, as well as the net average delays across all travelers.

The model developed for this purpose is a straightforward extension of those already presented. It is described in Section 5.1. Numerical analysis is presented in Section 5.2.

5.1 Car Delays under Holding
The range of bus arrival times that are suitable for holding was determined in Gu et al (2013) by the following inequalities:

\[ t_a \geq \frac{d}{w}; \quad (14) \]

\[ t_a \leq \frac{(1-g)q}{Q-q} L_c; \quad (15) \]

\[ t_a + S_{MAX} \leq L_c + \frac{d}{w}; \quad (16) \]

\[ t_a + S_{MIN} \geq t_q. \quad (17) \]

Inequality (14) means that a bus can reach a near-side stop only when the stop is not engulfed by a car queue. Inequality (15) indicates that the bus will impede the discharging car flow upon its arrival to the stop, so that the deferment of its arrival would benefit cars. Inequalities (16) and (17) specify that, as long as \( S \in [S_{MIN}, S_{MAX}] \), the bus will incur post-loading delay because it will finish loading/unloading passengers when it is still engulfed by a car queue formed during the red phase. Under these conditions, deferring a bus arrival by a suitable duration will not delay the bus’ departure from the intersection. The \( t_q \) in (17) is given by:

\[ t_q = \begin{cases} \frac{dQ}{wQ_B} + gL_c, & \text{if } \frac{d}{w} \leq t_a \leq \frac{q-gQ_B}{Q-Q_B} L_c - \frac{dQ(Q_B-q)}{wQ_B(Q-Q_B)} \frac{Q}{Q_B} t_a, \\ \frac{dQ}{wq} + \left( g \frac{Q_B+q}{q} - 1 \right) L_c + \frac{Q-Q_B}{q} t_a, & \frac{q-gQ_B}{Q-Q_B} L_c - \frac{dQ(Q_B-q)}{wQ_B(Q-Q_B)} \leq t_a \leq \frac{q-gQ_B}{Q-Q_B} L_c. \end{cases} \quad (18) \]

Suppose for illustration that the shaded time-space region in Fig. 15 is enclosed by the above inequalities, where: segment OR is from (16); MN and RM are determined from (17);\(^9\) and (14) and (15) are slack constraints in this example. Consider too the bus arrival marked by the “X” in the figure. Because its arrival time falls within the shaded region, the bus is eligible for holding: the horizontal distance between the X and the shaded region’s right-side border is the maximum duration that the bus can be deferred from reaching the stop without delaying its departure from the intersection; i.e. without delaying the bus over the longer run. We denote this latest allowable arrival time as \( t_a^{H} \). Consideration of the geometry in Fig. 15 shows that:

\(^9\) Inequality (15) is slightly different from the model in Gu et al (2013), because the latter sought to minimize the residual car queue, while (15) seeks to minimize car delay.

\(^{10}\) Note that (17) can mean two borders of the region because \( t_q \) can take one of the two expressions in (18), depending upon conditions.
\[ t_{a H} = \begin{cases} \min \left\{ \frac{(1-g)q}{q-q} L_C, L_C + \frac{d}{w} - S_{\text{MAX}}, \frac{q S_{\text{MIN}} - \frac{dQ}{w} (gQ_B - (1-g)q)L_C}{q - Q_B - g} \right\}, \\
\text{if } \frac{Q_B}{Q-B} - \frac{dQ}{wQ_B} (Q_B - Q) \leq t_a \leq \frac{Q_B}{Q-B} - \frac{dQ}{wQ_B} (Q_B - Q) && \text{and } Q + Q_B - Q \leq 0, \\
\min \left\{ \frac{(1-g)q}{Q-q} L_C, L_C + \frac{d}{w} - S_{\text{MAX}} \right\}, \\
\text{otherwise} \end{cases} \] (19)

Fig. 15 The region of bus arrivals selected for holding

To estimate the effect of holding on the expected added car delay, each \( t_a \) that satisfies (14) – (17) can be replaced with \( t_{a H} \) as calculated in (19). Equations (2), (3) and (5) of Section 2.1 can then be used to obtain \( E(W_C) \) as previously described. Since holding does not affect bus departure times from the intersection, \( E(W_B) \) is invariant to whether or not the bus is held.

5.2 Analysis

For illustration, Figs. 16a and b present \( E(W_C) \) as functions of \( d \). The figures are identical to Figs. 9a and b of Section 4, except that the former include \( E(W_C) \) under bus holding; see the dash-dot curves in the new figures. Note how holding reduces \( E(W_C) \) significantly when \( d \) is small.11

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11 As \( d \) grows large, the number of buses suitable for holding diminishes, along with the benefits that holding can bring; see Gu et al (2013) for further discussion on this matter.
Visual inspection of Fig. 16a further reveals that when an approach is under-saturated, holding can make a near-side stop all the more preferable to cars for $d < 100m$. Fig. 16b shows that holding can even make near-side stops preferable to cars on a near-saturated approach.

The benefits can translate to net savings in traveler delays as well. Fig. 17 is a reproduction of Fig. 13, except that the former presents the expected traveler delay under holding as well. The new figure shows that when the approach is under-saturated, bus-holding can render the near-side stop the best option for travelers, even for small $d < 100m$.

The benefits of holding diminish rapidly when the variation in bus dwell time, $\Delta S$, increases. Under these circumstances, selecting suitable candidates for holding becomes a more risky proposition. Fig. 18 presents an illustrative case in which $\Delta S = 40s$, and all other parameters are as in Fig. 16a. Notice how holding in these instances brings no benefits, irrespective of $d$.

Fig. 16 Comparisons of expected car delays at near-side (with and without holding) and far-side stops where $Q = 1, Q_B = 0.5, S_{MIN} = 50, S_{MAX} = 70$: (a) $\frac{q}{Q} = 0.4$; (b) $\frac{q}{Q} = 0.47$

Fig. 17 Expected passenger delays when holding is considered, where $\frac{q}{Q} = 0.4, \Delta S = 20s, O_B = 40$, and $O_C = 1.5$
6. Conclusions

The present models predict the expected delays that a dwelling bus can impart to vehicles and their occupants. These models were developed for cases in which (i) a curbside bus stop is to be placed on or downstream of an under- or near-saturated approach to a signalized intersection; and (ii) buses will occupy one of multiple lanes while dwelling at that stop. The models can be used to determine where to place the stop to suit local conditions. Our own parametric analysis points to some rough guidelines in this regard. These are summarized below.

If, as is often the case, a bus agency wishes to place the stop in the immediate vicinity of the intersection (i.e. $d \approx 0$), a far-side stop tends to shorten the delay imposed on the bus. This will often translate to a smaller net delay to travelers (i.e. people) as well.

In contrast, near-side stops tend to shorten the delays collectively incurred by cars if the stop can be placed some distance from the intersection, or if bus holding is deployed. The benefit can translate to shorter traveler delay. Large variations in bus dwell time can negate the benefit of holding, however. And if this variation is small yet holding is not viewed as a feasible option, then a far-side stop can be the preferred option for cars and people when the intersection is nearly-saturated, or when bus dwell time is long.

Our models apply to cases of uniform car inflows, which may occur if the bus stop’s neighboring signal is isolated from other traffic signals. Happily, the present methods can also be used to predict delays when car inflow is not uniform over time. In cases of the latter, guidelines might be drawn that are distinct from those given above. For instance, car and bus arrivals to an intersection and its neighboring bus stop are often consolidated by another traffic signal at an intersection further upstream. If the green phase of the upstream signal is perfectly coordinated with that of the stop’s neighboring signal, a far-side stop will always be preferred over a near-side one. Examples of this perfect-coordination case are shown.

Fig. 18 Comparisons of expected car delays at near-side (with and without holding) and far-side stops with where $Q = 1, Q_B = 0.5, S_{MIN} = 40, S_{MAX} = 80$ and $\frac{q}{Q} = 0.4$. 
in Figs. 19a and b for a pair of near- and far-side stops, respectively. Comparison of these two time-space diagrams reveals that: (i) a bus dwelling at a near-side stop can be impeded by the car queue created by the bus itself, while at a far-side stop the bus would experience no added delay; and (ii) the bus imparts smaller queues and delays to cars if it dwells at a far-side stop. The latter is true because, at both stop types, the dwelling bus will create the same residual car queues at the end of the green phase during which the bus arrives (see the bold, dashed lines in Figs. 19a and b). However, at the far-side stop a portion of this residual queue can discharge during the red phase, while all those cars queued at the near-side stop would have to wait until the next green phase to discharge.

Things are different when the two signals are not coordinated. Figs. 19c and d show an extreme case in which batched cars that discharge from the upstream signal arrive at the downstream intersection during its red phases. It is seen that the near-side stop is better off in this extreme case. This is because i) a bus dwelling at a near-side stop can load and unload passengers when it is engulfed by a car queue, which reduces both the bus delay and the impedance to cars; and ii) cars queued between a bus dwelling at the near-side stop and the downstream intersection can always discharge at maximum rate $Q$.

Ongoing work is directed at refining the present models to accommodate short block lengths, where a bus stop has nearby signalized intersections both upstream and downstream of it. We also intend to use the present models to assess traffic signal strategies that prioritize bus movements.

**Acknowledgements**

Funding for this work was provided by the Volvo Research and Educational Foundations and the University of California Transportation Center.
Fig. 19 Time-space diagrams when car arrivals are batched: (a) a near-side stop in the perfect-coordination case; (b) a far-side stop in the perfect-coordination case; (c) a near-side stop in the no-coordination case; (d) a far-side stop in the no-coordination case.
References


